Advanced modelling of road toll schemes

Authors: ***François Barbier-Saint-Hilaire**, INRETS,* ***Markus Friedrich**, PTV,* ***Ingmar Hofsäß**, PTV,* ***Wolfgang Scherr**, PTV*

Prediction of price elasticity is fundamental in the planning of toll financed highways and in the design of road toll schemes. Conventional assignment models use a value of time that is constant over all individual network users. This assumption leads to a crucial simplification in path choice, as time is globally converted into cost – the monocriterion approach This paper presents an advanced bicriterion assignment method called TRIBUT that allows a specific value of time to be attributed to each individual network user.

INTRODUCTION

TRIBUT is available within VISUM, the network model within the planning software PTV-Vision [3,4]. TRIBUT-VISUM is the result of a two year software development period performed by INRETS, the French Institute for Transport Research, together with the German software company PTV. TRIBUT is originally a result of French research [1,2] and since the early 1990's, it had been used by French transport engineers in numerous toll projects throughout the world.

The paper firstly will describe the general problem of assignment with time and cost. Then the main features of bicriterion path choice are presented, i.e. random distributed value of time and 'efficient' paths. The whole assignment algorithm is briefly presented. Finally, the main aspects of the TRIBUT application in practice will be discussed.

GENERAL PROBLEM FORMULATION: MINIMISING TIME AND COST

For toll assignments, the criteria for choosing path p consist of time tp and cost cp. The objective function or the generalised path choice criterion $Crit_p$ can be formulated as follows:

$$Crit_p = t_p + c_p / VT = \sum_{L \in p} t_L + \left[\sum_{L \in p} c_L \right] / VT$$

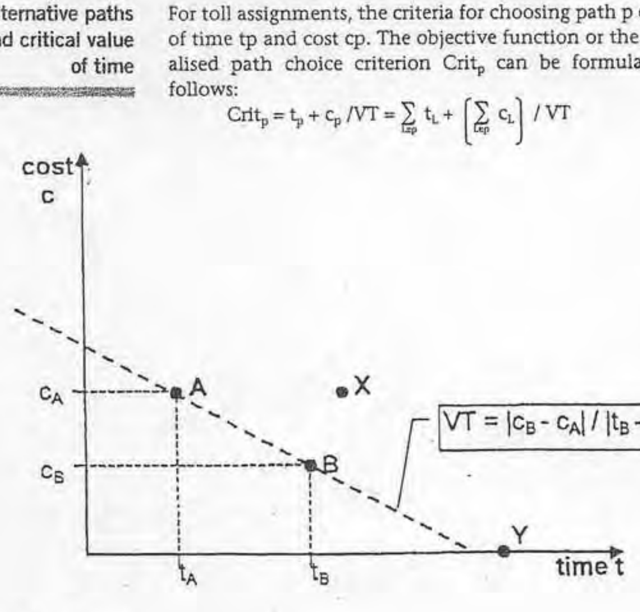

Figure 1: Time-cost diagram with alternative paths and critical value of time

with:

t_L travel time on a network object L as a function of traffic volume $t_L = t(vol_L)$,
L may represent a link, a node or a turning movement,
vol_L volume of link L – depending on link volume,
c_L toll for using link L – invariant of link volume,
VT value of time in [e.g. Euro/h] .

It is assumed, that each individual trip tries to minimise this criterion Critp in its path choice within a road network. So far the conventional monocriterion toll assignment and TRIBUT can be described by the same objective function. However, they differ in modelling the value of time VT:

- In the *monocriterion* case the value of time VT is assumed to be constant for all trips, or at least for all trips within one demand class. Therefore the expression c_p/VT in the objective function $Crit_p$ represents a constant supplement to time t_p for each path p. As a consequence the value of $Crit_p$ is identical for all trips.
- In the *bicriterion* case of TRIBUT the value of time VT is randomly distributed. That way each trip within a matrix can apply a specific value of time and the fact that each individual traveller has his own preferences about spending time and money is taken into account .

The time-cost diagram in Figure 1 illustrates some aspects of the bicriterion approach: alternative paths for one o-d pair are represented as a set of points, each characterised by time and cost, e.g. path A = (t_A, c_A). A specific value of time VT can be represented by any straight line with the slope -VT. If two paths fit the same VT-straight-line , they are considered indifferent, i.e. 'equally good' for a user who has this particular VT. On the other hand, the VT, which is determined by two alternative paths, is called "critical value of time" for the two alternatives in question. In Figure 1 the straight line representing the critical VT for A and B ($vt_{crit,A-B}$) is drawn as a dotted line. The point where the VT-line fits the cost-axis represents the total cost equivalent of both paths A and B, for the specific value of time. The corresponding point on the time-axis represents the general criterion (i.e. the time equivalent). It is obvious that path A will be preferred to X for any value of time, i.e. A dominates X. With VT = $vt_{crit,A-B}$ each path located on the right side of the VT-line is dominated by A and B (so is Y), because it is assumed that all trips prefer paths that minimise the general criterion as well as the cost equivalent.

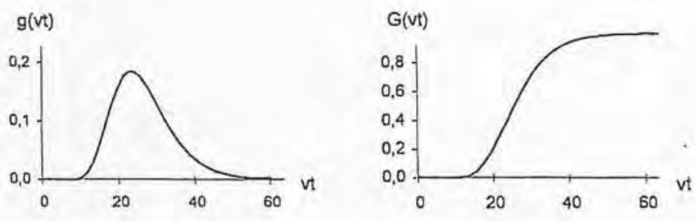

Figure 2: Density g(vt) and distribution function G(vt)= log N($\overline{VT}$,σ)

THE TRIBUT MODEL AND ALGORITHMS

Random Distributed Value of Time

As mentioned before, TRIBUT assumes that each vehicle applies its individual value of time VT. This assumption is reflected in the model by defining the value of time VT as a random variable with a distribution of the log-normal type:

VT = log N ($\overline{VT}$, σ)
with the following distribution parameters:

$\overline{VT}$ the median of VT = log N ($\overline{VT}$, σ)

σ the standard deviation of the associated random variable Y=$\log_e$(VT), where Y is normally distributed.

The logN-distribution is widely used in income statistics. One important property of the logN-distribution for income or toll modelling is that the probability equals zero for negative values, which is an essential assumption for values of time. The use of the median as positioning parameter may appear strange, but this corresponds to a convention in income statistics to publish quantils rather that mean values. Nevertheless the logN-distribution could as well be defined by the mean μ=$\log_e$(vt) and standard deviation σ both of the associated normal distribution Y=$\log_e$(VT).

Using TRIBUT in a planning project, the definition of the VT distribution, i.e. of the two parameters, is one of the most crucial steps. Later on, empirical methods to determine the distribution parameters will be discussed.

Determining the Efficient Alternatives in Path Search

In monocriterion methods one unique best path (shortest path) can be determined for each o-d pair. In the bicriterion case of TRIBUT however – due to the random distribution of VT – a unique 'best' path does not exist and numerous 'best paths' must be determined and stored in memory not only during path search but during the entire assignment procedure. That is the reason for the high complexity of multicriterion methods. Nevertheless, it is possible to reduce this complexity by identifying the efficient paths for each o-d pair.

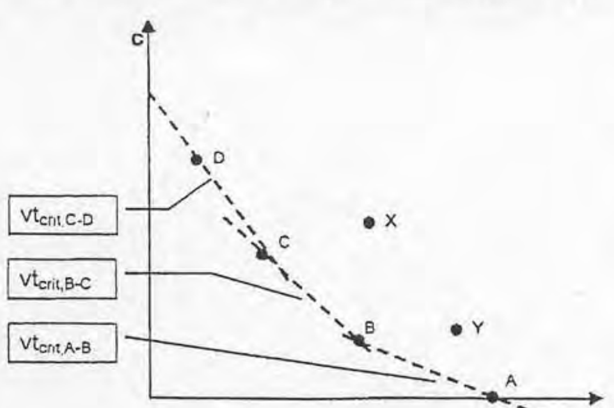

Figure 3 Critical values of time building up the efficient frontier

Figure 3 shows a path search with six paths, where paths A, B, C and D represent the set of efficient paths. It can easily be demonstrated either graphically or analytically that there is no VT for which paths X or Y would be preferred over A, B, C or D. Generally speaking, the convex curve formed by the three critical VT-straight-lines A-B, B-C and C-D limits the range of relevant cost-time combinations to the right side. This convex curve is called efficient frontier. Therefore the alternatives X and Y can be discarded in the example, because no traveller will choose them, if he minimises time and cost.

The introduction of the efficient frontier has important consequences:

- Only the efficient paths need to be stored during path search and path choice. As a consequence the majority of the various possible paths for one o-d pair can be discarded thus limiting computing time and memory.
- Nevertheless TRIBUT needs to perform a multi-path search that is more complex than a best-path-search in conventional monocriterion assignments.
- The set of efficient paths of any o-d pair is unique for one state of the network, meaning that it does not depend on the kind of VT-distribution which has been defined. Hence in the case of a multi-class assignment, only one search step has to be performed in each equilibrium iteration, although the subsequent demand allocation will be determined specifically for each demand class.

Path Choice for a Given Set of Alternatives

The path choice aims to allocate travel demand of an o-d pair onto the set of efficient paths. This allocation depends on the critical values of time between pairs of adjacent paths along the efficient frontier. In our example, there are three critical VTs defined by the pairs A-B, B-C and C-D. The demand share of each efficient path is deduced from the given probability distribution function.

In Figure 4 the distribution function is evaluated for the three values VT=$vt_{crit,A-B}$, VT=$vt_{crit,B-C}$ and VT=$vt_{crit,C-D}$. The share P(A) of alternative A on the lowest cost level is :

P(A) = G (VT = $vt_{crit,A-B}$).
The shares of B, C and D are:
P(B) = G (VT = $vt_{crit,B-C}$) - P(A),
P(C) = G (VT = $vt_{crit,C-D}$) - G(VT=$vt_{crit,B-C}$) and
P(D) = 1 - G (VT = $vt_{crit,C-D}$)

This allocation states that trips, performed by travellers who are not at all willing to spend money on travel (VT$\approx$0), up to trips with a VT <= $vt_{crit,A-B}$ will definitely choose the cheapest path A. Trips on the more 'wealthy' side of the travel demand with VT >= $vt_{crit,C-D}$ will choose the fastest and most expensive path D.

The Equilibrium Iteration in TRIBUT

Within the TRIBUT assignment the direct path choice given in Figure 4. is applied only once to provide an initial loading. But as travel time on network objects (links, nodes) is flow-dependent, the path choice for one specific o-d pair depends on the path choice of all other o-d pairs. This leads to an iterative procedure attempting to find a solution, where all o-d pairs are in equilibrium state.

TRIBUT in VISUM fulfils the Wardrop-Law. Where most other assignment software attempt an Wardrop-equilibrium for the whole network system, TRIBUT-VISUM determines it for each o-d. An o-d pair is in an equilibrium state, if the following conditions are accomplished:

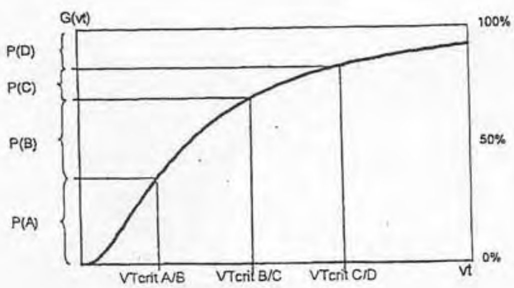

Figure 4: Path choice for initial loading

- path search does not find other efficient paths for the o-d pair,
- flow-dependent travel time is identical for all efficient paths on the same cost level,
- the shares of demand on the different cost levels correspond to the VT-distribution.

Figure 5 (see overleaf) presents a simplified flow chart of the TRIBUT assignment. Note, that flow-dependent travel times on links and on paths are calculated at several points in this procedure. A multi-path-search is performed at the beginning of each new assignment iteration. If new efficient paths are found, they are added to the set of existing efficient paths. VISUM stores all path-information (itinerary, used network objects, allocated demand), so that complete path information is available during the whole assignment process as well as after assignment for post-assignment analysis. If new paths have been found for a specific o-d pair, the o-d demand must be reallocated within the resulting new efficient frontier to obtain a new equilibrium state. This is achieved in two steps: an INTRA-Level-Balancing (reallocation of demand among the paths of the same cost level) and INTER-Level-Balancing (reallocation of demand between the paths of two adjacent cost levels).

The demand allocation (inter- and intra-level-balance) shifts demand from one efficient path to another. Each shift changes paths' travel times, as they depend on the traffic volumes. That way, the shape of the efficient frontier is continuously modified. Figure 6 illustrates an efficient frontier with three paths (A, B, C). The new path N is found, lying on the same cost level as B. So demand is shifted from B to N (intra-level-balance). As a consequence travel time t_B and t_N change. That way both paths will move to a point in the middle of their initial positions. As a result the slopes $vt_{crit,A\text{-}B}$ and $vt_{crit,B\text{-}C}$

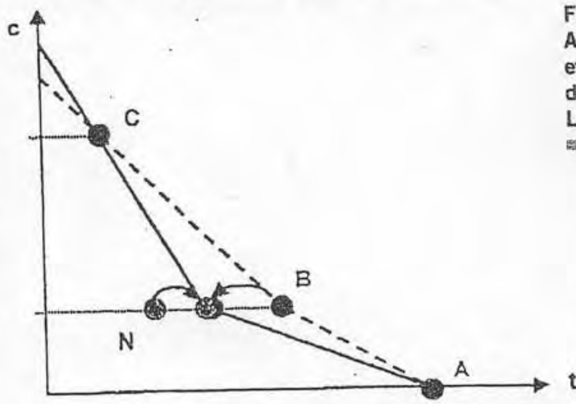

Figure 6: Adjustment of the efficient front during INTRA-Level-Balancing

change. So demand must be reallocated again among all efficient paths and between the different cost levels (inter-level-balance).

The complexity of the bicriterion assignment is higher than in conventional assignments, because of the multi-path-search and the total number of paths to be stored. Additionally both criteria t_R and c_R need to be accessible and to be stored for each path. But during first applications of VISUM-TRIBUT for French and German toll road planning the use of computing time and memory space could be optimised, such that TRIBUT runs well for large network models on personal computers.

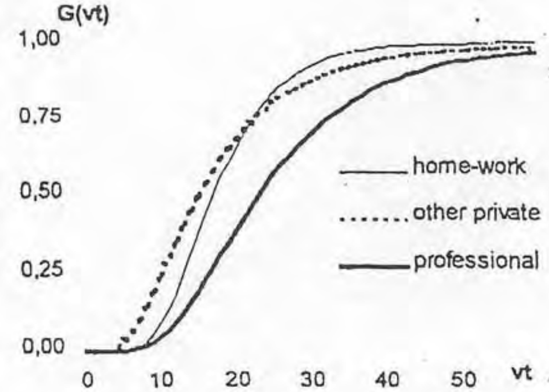

Figure 7: Multi-class VT-distributions

THE APPLICATION OF TRIBUT IN PRACTICE

Toll schemes: VISUM allows to define linear and non-linear road pricing schemes. Linear toll is modelled as a toll value per road segment (link). To model classical toll schemes, where toll is paid for the next road section at stations which are located at entry or exit points, a linear toll model is sufficient. In modern toll projects, more sophisticated systems of pricing are applied. Especially telematic techniques allow to design price schemes, where the price to drive from A to C via B may not be equal to the sum of A-B and B-C. These non-linear toll schemes can be modelled within VISUM as a price matrix between motorway entries and exits.

Multi-class models: VISUM provides multi-class assignment, where several demand classes with specific o-d matrices can be assigned simultaneously. The introduction of demand classes to the model is helpful in the case of group specific pricing for different vehicle types or travellers with different types of tickets (single riders, commuter pass holders). Additionally specific VT-distributions can be defined for each demand class thus leading to a more realistic composition of the entire demand and of its elasticity to pricing policies. An example of user classes for home-work-commuters, other private demand and professional demand is shown in Figure 7.

Figure 5: Flow chart of the equilibrium iteration

```
** INITIAL LOADING: **
FOR origins o := 1 ... N DO
    Perform MULTI-PATH-SEARCH to all destinations d := 1 ... N
    FOR all destinations d := 1 ... N DO
        Allocate o-d-trip-demand to the efficient paths of o-d
        Store the efficient paths of o-d
    Compute flow dependent travel times for all network objects of the efficient path set
** EQUILIBRIUM ITERATION: **
WHILE NOT all-od-in-equilibrium DO
    all-od-in-equilibrium := TRUE
    FOR origins o := 1 ... N DO
        Perform MULTI-PATH-SEARCH to all destinations d := 1 ... N
        IF new efficient path are found THEN o-in-equilibrium := FALSE
        Store paths
        WHILE NOT o-in-equilibrium DO
            o-in-equilibrium := TRUE
            FOR all destinations d := 1 ... N DO
                pod-in-equilibrium DO
                od-in-equilibrium := TRUE
                    FOR all cost-levels of o-d DO
                        INTRA-Level-Balance
                        Compute travel times for all network objects
                    FOR all neighbour-cost-levels of o-d DO
                        INTER-Level-Balance
                        Compute travel times for all network objects
                    IF  flow has been changed on any network object THEN
                        od-in-equilibrium := FALSE
                        o-in-equilibrium := FALSE
                        all-od-in-equilibrium := FALSE
```

Finding the VT parameters: When TRIBUT is applied in a planning project, there are three ways to determine the parameters VT and σ: revealed-preference surveys, stated-preference surveys or macroeconomic calculus. The weakest but cheapest method is macroeconomic calculus, which for instance divides the total work income of a society by the total work time in order to obtain a mean value of time for trips to work. This approach doesn't reflect behavioural differences e.g. between commuters in urban areas and long distance travellers. Secondly, this approach determines a mean value of time, which is less appropriate than quantils (medians).

When a toll is introduced into a society without revealed toll experience, usually stated-preference methods are applied. In the stated-preference interview different hypothetical situations with variation of time and cost are simulated to find the critical cost-time combination ('transfer price'), where the traveller changes his behaviour. Then, maximum likelihood estimation determines the two parameters. It has been found that people find it hard to imagine a toll situation if they are not used to tolls in real life. Consequently, the standard deviation σ is mostly overestimated by the stated preference method.

The best method is to estimate the value of time on the basis of revealed preferences. This requires a toll road and pricing system, which operates at least three or four years prior to the survey, to ensure that the system has come to an equilibrium. Recently a revealed preference study has been carried out in the area of Marseille/France [6]. The method consisted, first, of an o-d survey, measuring the shares of cars using the toll road or the alternative non-priced infrastructure, and secondly the measurement of real-travel time for each o-d at different day periods. The most important results are:

- Time saving was found to be the most explicable variable of the use of toll roads, no statistical effect was found for travel time deviations or road comfort.
- The median of the value of time is a very significant and stable value, even if different types of distributions are estimated.
- The logN parameters for the global demand were estimated by Vt = 9 Euro/h and $\sigma = 0.66$.

159

TRAFFIC ENGINEERING & CONTROL
tec
The International Journal of Traffic Management and Transportation Planning

32 Vauxhall Bridge Road,
London, SW1V 2SS

Telephone: 020 7973 6400
Editorial Fax: 020 7973 6677
Ad Dept Fax: 020 7233 5053
Marketing Fax: 020 7233 5052

Different trip purposes show a less significant influence on the value of time parameters than the fact whether a traveller must pay the toll 'out of his own pocket' or whether he gets it refunded, for instance by his company.

Travel time modelling: As cost is not flow-dependent, it is a given model input. On the other hand, time is modelled by the help of flow-dependent functions and therefore represents a less certain input of the assignment model. But the quality of the forecasted volumes in toll projects depends on travel time as well as on cost. Therefore the modelling of the flow-dependent time on links and nodes requires more attention than in ordinary planning projects without toll:

For trips which originate or terminate outside the model area (e.g. transit), the total travel time can't be evaluated by the network model and thus it is not recommended to apply the same value of time distribution as for internal trips.

Link and node flows that exceed capacity should be avoided, as in this case capacity restraint functions do not produce realistic travel times. Especially if peak hours are modelled, the capacities of highly charged links need to be defined very carefully.

CONCLUSION

TRIBUT has been used for several road toll studies especially within urban areas throughout Europe, America and Australia. With its implementation in VISUM, TRIBUT is available for practitioners on a multi-language platform under a Windows interface. Based on the experience of successful studies, TRIBUT is widely accepted by road financing companies as a basis for establishing realistic revenue forecasts and by planning authorities to estimate the impacts on the complementary network. The advantages of the TRIBUT method become obvious when different pricing policies need to be tested. The conventional monocriterion approach turned out to be inappropriate to model real price elasticity. Recent empirical research in France confirms the bicriterion approach and the log-normal-distributed value of time.

References

1 Barbier-Saint-Hilaire, F., Traitement des péages dans DAVIS. Paper for DAVIS user meeting, Argueil/ France 1996.

2 Leurent, F., Cost versus Time Equilibrium over a Network. Presented at the 1994 Annual Conference of the Transportation Research Board, Washington D.C. 1994.

3 Friedrich, M., A Multi-Modal Transport Model for Integrated Planning. In Book of Abstracts of 8th World Conference on Transport Research, Antwerp 1998.

4 Fellendorf, M., Haupt, Th., Heidl, U., Scherr, W., PTV VISION: Activity-based demand forecasting in daily practice. In Activity-Based Approaches to Travel Analysis, p. 55-72, Elsevier/Oxford 1997.

5 Dial, R.: Network-Optimised Road Pricing: Part I: A Parable and a Model. Part II: Algorithms and Examples. In: Operations Research, vol. 47, No. 1+2, 1999.

6 Maresca, C., Delons, J., Leurent, F., Comportements des automobilistes face au péage urbain. L'expérience du tunnel Prado-Carénage. Collections dossiers du CERTU, Lyon/France 1999.

About the authors

INRETS (Institut National de Recherche sur les Transports et leur Sécurité): 2, av. du Gén. Malleret-Joinville, F-94114 Arcueil, France
Phone +33/1/47407100 email bsh@inrets.fr

PTV Planung Transport Verkehr AG:
Stumpfstr. 1, D-76131 Karlsruhe, Germany
Phone +49/721/9651/200, email markus.friedrich@ptv.de or wolfgang.scherr@gmx.net